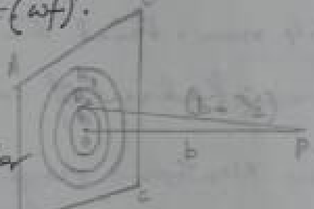
DIFFRACTION OF LIGHT

When light from a narrow linear slit is incident on the sharp edge of an obstacle, it is found that there is illumination to some extent within the geometrical shadow of the obstacle. It shows that light can bend round an obstacle.

Such effects of bending of light round corners and encroachment of light within the geometrical shadow of an opaque obstacle are called diffraction of light. The effect is found to be significant when the dimension of the diffracting element becomes comparable with the wavelength of light.

FRESNEL'S HALF-PERIOD ZONES OF A PLANE WAVEFRONT:-

Fresnel gave an explanation of the phenomena of diffraction of light on the basis of the mutual interference of the secondary waves of wavelets from the various points of a wavefront (wf).



Consider a plane wavefront wf ABCD of monochromatic light (wavelength λ) advancing to the right and perpendicular to the plane of the paper. To find the resultant disturbance at P due to all the wavelets coming from every points of the wf, the whole wf is divided into a no. of Fresnel half-period zones (hpz). From P, a perpendicular $PO = b$ is drawn on the wf meeting it at O which is called the pole of the wave w.r.to P. With P as centre and radii $(b+\lambda/2)$, $(b+2\lambda/2)$, $(b+3\lambda/2)$ ... etc. spheres are drawn, the sections of which by the plane of the wf are concentric circles H_1, H_2, H_3 etc. The area enclosed by circle H_1 is called first half-period zone. The annular zone, between the circles H_1 and H_2 is called second half-period zone and so on.

Areas of zones :-

The area of the n^{th} zone, i.e. the area b/w the circles H_n and H_{n-1} is,

$$A_m = \pi\left\{(PH_m^2 - b^2) - (PH_{m-1}^2 - b^2)\right\}$$

$$= \pi\left\{\left(b + m\frac{\lambda}{2}\right)^2 - b^2\right\} - \pi\left[\left\{b + (m-1)\frac{\lambda}{2}\right\}^2 - b^2\right]$$

$$= \pi b\lambda + \pi(2m-1)\frac{\lambda^2}{4} \quad \text{---(1)}$$

$$\simeq \pi b\lambda \qquad [\text{assuming } b \gg \lambda]$$

Let $a_1, a_2, \ldots a_m$ be the resultant amplitudes at P due to all the wavelets coming from 1st, 2nd, ... m^{th} zones resp. Hence the resultant amplitude at P due to all the zones is, $R = a_1 + a_2 + \ldots + a_m$.

According to Fresnel, the amp. at P due to wavelets coming from the m^{th} zone depends on following factors:

(i) $a_m \propto A_m$ (area of m^{th} zone)

(ii) $a_m \propto \dfrac{1}{d_m \text{ (dist. of } P \text{ from } m^{th} \text{ zone)}}$

(iii) $a_m \propto$ obliquity factor, $f(\theta_m)$.

where θ_m is the angle which the dir^n of P from the m^{th} zone makes with OP. The factor $f(\theta_m)$ is 1 for $\theta_m = 0$ and tends to zero as θ_m tends to $90°$. $[f(\theta_m) \sim \cos\theta]$

now d_m (mean dist. of m^{th} zone from O) is

$$= \frac{1}{2}\left[b + \frac{m\lambda}{2} + b + (m-1)\frac{\lambda}{2}\right] = b + \left(m - \frac{1}{2}\right)\frac{\lambda}{2} \quad \text{---(2)}$$

now $$\frac{A_m}{d_m} = \frac{\pi b\lambda + \pi\left(m - \frac{1}{2}\right)\frac{\lambda^2}{2}}{b + \left(m - \frac{1}{2}\right)\frac{\lambda}{2}} = \frac{\pi\lambda\left[b + \left(m - \frac{1}{2}\right)\frac{\lambda}{2}\right]}{b + \left(m - \frac{1}{2}\right)\frac{\lambda}{2}} = \pi\lambda$$

[From (1) & (2)]

Since, $\frac{A_m}{d_m} = \pi\lambda$ (constant), therefore the successive amplitudes $a_1, a_2, a_3 \ldots$ etc will be in descending order of magnitude due to the obliquity factor.

Since a given zone is $\lambda/2$ farther away from P than the previous one, the disturbances from alternate zones will have opposite phase at P. Hence,

$$R = a_1 - a_2 + a_3 - a_4 + \ldots + a_m$$

or, $$R = \frac{a_1}{2} + \left(\frac{a_1 + a_3}{2} - a_2\right) + \left(\frac{a_3 + a_5}{2} - a_4\right) + \ldots + \frac{a_m}{2} \quad [\text{if } m \text{ is odd}]$$

and $$R = \frac{a_1}{2} + \left(\frac{a_1 + a_3}{2} - a_2\right) + \left(\frac{a_3 + a_5}{2} - a_4\right) + \ldots + \frac{a_{m-1}}{2} - a_m \quad [\text{if } m \text{ is even}]$$

As $a_1, a_2, a_3 \ldots$ are in descending order of magnitudes, $\frac{a_1 + a_3}{2} \simeq a_2$ and so on. Hence all terms within the brackets in above equations cancel out and so we get,

$$R = \frac{a_1}{2} + \frac{a_m}{2} \quad [\text{when } m \text{ is odd}]$$

$$R = \frac{a_1}{2} + \frac{a_{m-1}}{2} - a_m \quad [\text{when } m \text{ is even}]$$

For very large 'm', greater obliquity of zones causes a_{m-1} and a_m vanishingly small and hence $R = \frac{a_1}{2}$.

Thus the resultant amplitude at P due to the whole wavefront is equal to half the amplitude of the secondary waves from the first half-period zone.

Explanation of rectilinear propagation of light :-

If the first hpz be covered by a small circular opaque obstacle then the resultant amplitude at P becomes $(a_2/2)$ and the intensity becomes proportional to $(a_2/2)^2$. By increasing the size of the obstacle, if 2^{nd}, 3^{rd} etc. hp zones are covered one by one, then the intensity of illumination at P will be proportional to $(a_3/2)^2$, $(a_4/2)^2$ etc. The wavelength of light being very small, the area of each zone is also very small, so a very tiny obstacle placed in the path of light is capable of cutting off a large no. of innermost effective zones and point P will be a dark point. Thus the law of rectilinear propagation of light is not rigidly correct, it is only an approximation.

ZONE PLATE :-

A zone plate is an optical device (special diffracting obstacle) designed to block off the light from alternate half-period zones.

The n^{th} zone on a plane wf has a radius,

$$r_n = \sqrt{\left(b + \frac{n\lambda}{2}\right)^2 - b^2} = \sqrt{nb\lambda} \quad [\because \lambda^2 \text{ is very small neglected}]$$

So, $r_1 = \sqrt{1.b\lambda}$, $r_2 = \sqrt{2.b.\lambda}$ and so on

Radii of different hp zones are proportional to the square root of all natural numbers.

Construction :—

To construct a zp it is necessary to draw on a white sheet of unglazed paper, large no. of concentric circles, with radii proportional to the square roots of natural nos. and then to blacken alternate zones. A reduced photograph on glass plate of this drawing constitutes a zp.

Now a plane wf. incident on zp will allow secondary wavelets to pass through alternate zones whereas the other zones will not allow them to pass through. ∴ The resultant amp. at an external point P w.r.to which the zones behave as hpzones will be,

$R = a_1 + a_3 + a_5 + \dots\dots$, which is evidently much bigger than the amp. $(a_1/2)$ that would have been produced had all the zones on the plate been transparent. As a result the pt. P will be greatly illuminated. This behaviour of zp is similar to that of a converging lens which also focusses a plane wf. Thus a zp behaves as a convex lens for parallel rays.

Similarity with a convex lens :—

Here 'S' is a bright pt. object sending out spherical wf's of wavelength λ and effects of the wf are to be determined at 'O'. Let PM_n be an imaginary plane $\perp^r$ to SO & also $\perp^r$ to the plane of the paper. Drawing concentric circles with P as centre and radii $PM_1 = r_1$, $PM_2 = r_2 \dots\dots PM_n = r_n$ etc. on the plane divides the plane into several zones. The points $M_1, M_2, \dots\dots M_n$ etc. are such that,

$$SM_1 + M_1O = SP + OP + \lambda/2$$
$$SM_2 + M_2O = SP + OP + 2\lambda/2$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$SM_n + M_nO = SP + OP + n\lambda/2$$

The area of the circle of radius PM_1 on the plane is called 1st hpz, annular space b/w circles of radii PM_2 and PM_1 is 2nd hpz and so on.

$$\left\{ SP = u \,;\; OP = v \quad PM_n = r_n \right.$$

$$SM_n = \left\{(SP)^2 + (PM_n)^2\right\}^{1/2}$$
$$= (u^2 + r_n^2)^{1/2}$$
$$= u\left(1 + r_n^2/u^2\right)^{1/2}$$
$$\simeq u + r_n^2/2u$$

Similarly, $OM_n = (v^2 + r_n^2)^{1/2} \simeq \left(v + \frac{r_n^2}{2v}\right)$

So, $SM_n + M_nO = SP + OP + n\lambda/2$ becomes,

$$u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} = u + v + n\frac{\lambda}{2}$$

$$r_n^2\left(\frac{1}{u} + \frac{1}{v}\right) = n\lambda$$

or, $$\boxed{\frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2}}$$

Above eq^n is similar to the real image eq^n $\boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$ of a convex lens. Comparing resp.,

$$\boxed{f = \frac{r_n^2}{n\lambda}}$$