FILTERS

Introduction

There are mainly three considerations in designing a filter circuit they are:

- The response of the pass band must be maximum flatness.
- There must be a slow transition from pass band to the stop band.
- The ability of the filter to pass signals without any distortions within the pass band.

These distortions are generally caused by the phase shifts of the waveforms. In addition to these three the rising and falling time parameters also play an important role. By taking these considerations for each consideration one type of filter is designed. For maximum flat response the Butterworth filter is designed. For slow transition from pass band to stop band the Chebyshev filter is designed and for maximum flat time delay Bessel filter is designed.

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Butterworth Filter

At the expense of steepness in transition medium from pass band to stop band this Butterworth filter will provide a flat response in the output signal. So, it is also referred as a maximally flat magnitude filter. The rate of falloff response of the filter is determined by the number of poles taken in the circuit. The pole number will depend on the number of the reactive elements in the circuit that is the number of inductors or capacitors used in the circuits.

The amplitude response of n^{th} order Butterworth filter is given as follows:

$$V_{out} / V_{in} = 1 / \sqrt{1 + (f / f_c)^{2n}}$$

Where 'n' is the number of poles in the circuit. As the value of the 'n' increases the flatness of the filter response also increases.

'f' = operating frequency of the circuit and 'f_c' = centre frequency or cut off frequency of the circuit.

These filters have pre-determined considerations whose applications are mainly at active RC circuits at higher frequencies. Even though it does not provide the sharp cut-off response it is often considered as the all-round filter which is used in many applications.

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Butterworth Approximations

As we know that to meet the considerations of the filter responses and to have approximations near to ideal filter we need to have higher order filters. This will increase the complexity. We know the output frequency response and phase response of low pass and high pass circuits also. The ideal filter characteristics are maximum flatness, maximum pass band gain and maximum stop band attenuation.

To design a filter, proper transfer function is required. In order to satisfy these transfer function mathematical derivations are made in analogue filter design with many approximation functions. In such designs Butterworth filter is one of the filter types. Low pass Butterworth design considerations are mainly used for many functions. Later we will discuss about the normalized low pass Butterworth filter polynomials.

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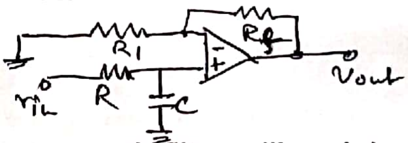
First Order Low Pass Butterworth Filter

The below circuit shows the low pass Butterworth filter:



The required pass band gain of the Butterworth filter will mainly depends on the *resistor values of* 'R1' and 'Rf' and the cut off frequency of the filter will depend on R *and C elements* in the above circuit.

The gain of the filter is given as A_{max} = 1 + (R1 / Rf)

The impedance of the capacitor 'C' is given by the $-jX_c$ and the voltage across the capacitor is given as,

$$V_c = - jX_c / (R - jX_c) * Vin$$

Where $X_c = 1 / (2\pi fc)$, capacitive Reactance.

The transfer function of the filter in polar form is given as

$$H(j\omega) = |Vout/Vin| \angle \phi$$

Where gain of the filter $V_{out} / V_{in} = A_{max} / \sqrt{1 + (f/f_H)^2}$

And phase angle Ø = - $\tan^{-1}$ (f/f_H)

At lower frequencies means when the operating frequency is lower than the cut-off frequency, the pass band gain is equal to maximum gain.

$$V_{out} / V_{in} = A_{max} \text{ i.e. constant.}$$

At higher frequencies means when the operating frequency is higher than the cut-off frequency, then the gain is less than the maximum gain.

$$V_{out} / V_{in} < A_{max}$$

When operating frequency is equal to the cut-off frequency the transfer function is equal to $A_{max} /\sqrt{2}$. The rate of decrease in the gain is 20dB/decade or 6dB/octave and can be represented in the response slope as -20dB/decade.

Butterworth Low Pass Filter Example

Let us consider the Butterworth low pass filter with cut-off frequency 15.9 kHz and with the pass band gain 1.5 and capacitor C = 0.001μF.

$$f_c = 1/2\pi RC$$

$$15.9 * 10^3 = 1 / \{2\pi R\} * 0.001 * 10^{-6}\}$$

$$\therefore R = 10k\Omega$$

$$A_{max} = 1.5 \text{ and assume R1 as } 10\ k\Omega$$

$$A_{max} = 1 + \{R_f / R_1\}$$

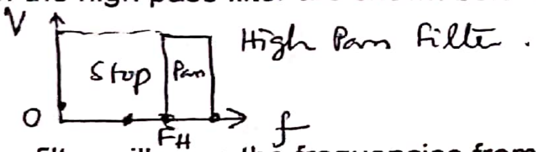
$$R_f = 5\ k\Omega$$

HIGH PASS FILTER

Introduction

A high pass filter will allow the frequencies which are higher than the cut-off frequency and attenuate the frequencies lower than the cut off frequency. In some cases this filter is also termed as 'Low-Cut' filter or 'Base-cut' filter. The amount of attenuation or the pass band range will depend on the designing parameters of the filter. The pass band gain of an active filter is more than unity gain. The operation of the active high pass filter is same as passive high pass filter, but the main difference is that the active high pass filter uses operational an amplifier which provides amplification of the output signals and controls gain.

The ideal characteristics of the high pass filter are shown below:



We know that the high pass filter will pass the frequencies from cut-off frequency point to 'infinity' frequency which does not exist in practical considerations. Besides passive high pass filter in this active high pass filter the maximum frequency response is limited by the open loop characteristics of the op-amp.

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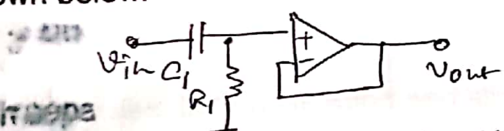
Active High Pass Filter

By connecting a passive RC high pass filter circuit to the *inverting or non-inverting* terminal of the op-amp gives us first order active high pass filter. *The passive RC* high pass filter circuit connected to the non-inverting *terminal of the unity gain* operational amplifier is shown below.



The gain $A_{max} = 1$ and cutoff frequency $f_c = 1/2\pi RC$

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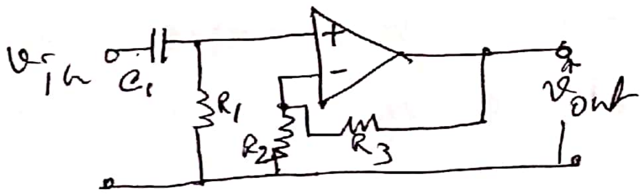
Active High Pass Filter with High Voltage Gain

The operation is same as that of the passive high pass filter, but the input signal is amplified by the amplifier at the output. The amount of amplification depends on the gain of the amplifier. The magnitude of the pass band gain is equal to 1 + (R_3/R_2). Where R_3 is the feedback resistor in Ω (ohms) and R_2 is the input resistor. The circuit of active high pass filter with amplification is given below.



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Voltage Gain of an Active High Pass Filter

$$\text{Voltage Gain } A_v = A_{max}\,(f/f_c) \,/\, \sqrt{1 + (f/fc)^2}$$

Where f = operating frequency

fc = cut-off frequency

A_{max} = pass band gain of the filter = $1 + (R_3/R_2)$

At low frequencies means when the operating frequency is less than the cut-off frequency, the voltage gain is less than the pass band gain A_{max}. At high frequencies means when the operating frequency is greater than the cut-off frequency, the voltage gain of the filter is equal to pass band gain. If operating frequency is equal to the cut-off frequency, then the voltage gain of the filter is equal to 0.707 A_{max}.

Voltage Gain in (dB):

The magnitude of voltage gain is generally taken in decibels (dB):

$$A_v(dB) = 20 \log_{10} (V_{out}/V_{in})$$

$$-3\,dB = \cancel{-3\,dB =} \; 20 \log_{10} (0.707 * V_{out}/V_{in})$$

The *cut-off frequency* which separates both pass band and stop band can be *calculated using the below formula*

$$f_c = 1 / (2\pi RC)$$

The phase shift of the active high pass filter is equal to that of the passive filter. It is equal to the +45° at the cut-off frequency f_c and this phase shift value is equated as

$$\varnothing = \tan^{-1}(1/2\pi f_c RC)$$

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Active High Pass Filter Example

Let us consider cut-off frequency value as 10 KHz, pass band gain A_{max} as 1.5 and capacitor value as 0.02 μF.

The equation of the cut-off frequency is $f_c = 1 / (2\pi RC)$

By re-arranging this equation,

we have $R = 1 / (2\pi fC)$

$$R = 1/ (2\pi * 10000 * 0.02 * 10^{-6}) = 795.77\ \Omega$$

The pass band gain of the filter is $A_{max} = 1 + (R_3/R_2) = 1.5$

$$R_3 = 0.5\ R_2$$

If we consider the R_2 value as 10KΩ, then $R_3 = 5\ k\Omega$

We can calculate the gain of the filter as follows:

Voltage Gain for High Pass filter:

$$| V_{out} / V_{in} | = A_{max} * (f/f_c) / \sqrt{[1 + (f/fc)^2]}$$

$$A_v(dB) = 20 \log_{10} (V_{out}/V_{in})$$

By using this equation let us tabulate the responses for the range of frequencies to plot the response curve of the filter. These responses are assumed as 10 Hz to 100 KHz.

Applications of Active High Pass Filters

- These are used in the loud speakers to reduce the low level noise.
- Eliminates rumble distortions in audio applications so these are also called are treble boost filters.
- These are used in audio amplifiers to amplify the higher frequency signals.
- These are also used in equalizers.